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*We describe the role of Lucas Bunt at the start of the teaching of probability and statistics in the last two years of Dutch secondary schools in the early 1950s. Together with his co-authors, Bunt developed an experimental text which, from the mid-1950s on, became a regular textbook. We further sketch Bunt's other – mostly international – activities with respect to the curriculum reform movement initiated at the Royaumont Seminar in 1959. Bunt's experiment can be seen as one of the initiatives related to this reform. Finally, we present what happened with statistics teaching in the Netherlands "after Bunt".*

*Keywords: Mathematics curriculum, Probability, Secondary school mathematics, Statistics.*

## Introduction

The attention to statistics in Dutch secondary school mathematics arose in the early 1950s when a student text about statistics was developed by a group of mathematics teachers led by Lucas Bunt. The text was used in experiments in the last two years of secondary schools that prepare students for the university, initially only for the non-exact streams of these schools. Bunt's reason to develop this text was a proposal made by *Liwenagel*, one of the two associations of teachers of mathematics at that time[1], to include statistics into the curriculum for these students. The proposal cannot be seen independently from the worldwide trend after World War II to include applications of mathematics into the secondary school curricula (De Bock & Zwaneveld, in press). During the 1950s the call for curriculum change was so strong that the OECD took the initiative to organize, in 1959, the Royaumont Seminar with representatives from different western countries to initiate the reform. Bunt attended "Royaumont" and many other international meetings related to this reform movement.

Although Bunt's pioneering role in statistics education is well-known in the Netherlands, a proper scientific review of his work is still missing. Moreover, his acting at the international math educational scene was not given appropriate attention so far, especially in the debates about a possible introduction of statistics at the secondary school level.

We present Bunt's role in the Dutch curriculum reform movement of the 1950s, more specifically, his activities related to the development of a statistics program as a part of it. Based on written historical sources and a few oral testimonies of contemporaries, we first provide some elements of Bunt's professional career. We then report about his experiment with the teaching of statistics in secondary school classrooms and about the actions he took to ensure that his ideas became

[1] The other organization of teachers of mathematics was *Wimecos*. Both organizations, *Liwenagel* and *Wimecos*, were the predecessors of the *Nederlandse Vereniging van Wiskundeleraren* [Dutch Association of Mathematics Teachers].

consolidated. Finally, we report what happened with statistics in Dutch secondary mathematics curricula "after Bunt" and we present some conclusions.

## Lucas Bunt

Lucas Nicolaas Hendrik Bunt (Figure 1) was born in 1905 in Edam, a small village north of Amsterdam. He studied mathematics at the University of Amsterdam where he also defended, in 1934, his PhD thesis, entitled *Bijdrage tot de theorie der convexe puntverzamelingen* [Contribution to the theory of convex point sets]. In the early 1930s, Bunt started his career as a mathematics teacher in Leeuwarden where he likely met his wife, a chemistry teacher at the same school. In the late 1940s Bunt became mathematics teacher trainer at the University of Groningen. From 1948 to 1969 he was appointed as a full-time mathematics teacher trainer at Utrecht University, a position that he combined with that in Groningen. In 1968, immediately after the retirement of his wife, he and his family migrated to Arizona (US) where Bunt became a professor of mathematics at Arizona State University. We assume that Bunt had already developed strong professional ties with the US in the early 1960s to secure this appointment, but could not verify this any further. Bunt died in 1984 in the US.

**Figure 1: Lucas Bunt (left: detail of a picture of the *Mathematics Working Group*, 1948; right: attending the public defense of the PhD thesis of his son Harry at the University of Amsterdam, 1981)**

Bunt became active at the math educational scene in the Netherlands shortly after World War II as a member of the *Mathematics Working Group*, a group that critically reflected on the existing secondary school curricula and developed proposals for new curricula (La Bastide-van Gemert, 2015). Bunt's international career started in 1959. Recommended by Hans Freudenthal to the Dutch Ministry of Education, Bunt was one of the three representatives for the Netherlands at the famous Royaumont Seminar and he co-edited the Seminar's Proceedings with Howard F. Fehr (OEEC, 1961). In the late 1960s, Bunt translated and adapted, in cooperation with Harrie Broekman, a series of booklets that were developed by the *School Mathematics Study Group* in the US. This resulted in a six-volume programmed instruction course for Dutch secondary school students.

Bunt was primarily a mathematician who explained mathematics to a non-mathematically schooled audience. We mention his textbook *Statistiek voor het voorbereidend hoger en middelbaar onderwijs* [Statistics for preparatory higher and secondary education] (1956), intended for Dutch students, aged 16 to 18 years, who prepared themselves for university studies in social sciences, economics, geography, etc., based on an experiment of which Bunt published the report (Bunt, 1957). For an international audience, Bunt (co-)authored *An introduction to sets, probability and hypothesis testing* (with Howard F. Fehr and George Grossman) (1964) and *Probability and hypothesis testing* (1968).

## First experiments with statistic education at the secondary level

Bunt took the initiative to develop an experimental text about statistics in some gymnasia A[2]. The text was initially mimeographed, in 1956 it was printed as a textbook (Bunt, 1956). As mentioned before, one of the reasons for Bunt to start with an experiment about the teaching of statistics was a proposal of a commission established by the organization of mathematics teachers *Liwenagel*, intended to study the opportunities and possibilities of "a re-organization of mathematics education in the A-streams of the gymnasia and the gymnasium sections of the lyceums" (1950). Bunt was a member of that commission and, although it is not mentioned, likely the main author of the commission's report.

It is worth mentioning that Bunt did not develop the experimental text and the textbook on his own, although this was a common practice in the Netherlands at that time, but in cooperation with a team of teachers. In the *Preface* of the textbook Bunt wrote (translated from Dutch):

> … was an educational experiment in statistics, organized by the Department of Didactics of the Pedagogical Institute of the State University of Utrecht. The following teachers cooperated: Dr. Cath. Faber-Gouwentak, Barlaeus-Gymnasium, Amsterdam; Sr. E. A. de Jong, *Rectrix* [Headmistress] St.-Theresia-Lyceum[3], Tilburg; D. Leujes, Grotius-Gymnasium, Delft; Dr. H. Mooy, Barlaeus-Gymnasium, Amsterdam; Dr. P. G. J. Vredenduin, *Co-rector* [Vice Headmaster] *Stedelijk* [Municipal] Gymnasium, Arnhem. (Bunt 1956, p. v)

At that time in the Netherlands, statistics was not a part of the official curriculum that only included algebra and geometry, topics that were also part of the final exams, organized centrally by the government. However, based on an exception rule, the Inspection of Education could allow teachers to change parts of the exam program. Such exception was obtained for the statistics experiment.

In 1957, Bunt published the report in which he describes the experiment with the student text that was used during the years 1951-1955 (Bunt, 1957). The reason why the textbook was published before this report was, as Bunt wrote in the textbook's *Preface*: "The recent proposals of the mathematics teachers associations *Wimecos* and *Liwenagel* about the curriculum change for mathematics in the B-stream of the secondary schools, in which statistics is included as a new topic, made it desirable to make, as soon as possible, the text public" (Bunt, 1956, p. v). Bunt's report has two parts: part A includes the motivation and explanation about the selected topics, and the way they are treated; part B is the student text (it is not included in the printed version of the report).

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[2] At that time, the gymnasia in the Netherlands had two study streams: The A-stream, preparing students for university studies such as languages, economics, psychology, sociology, history, and geography, and the B-stream, preparing students for university studies in mathematics, science and technology.

[3] A lyceum was a school for secondary education with two sections: gymnasium and *Hogere Burger School* (*HBS*) [Higher Citizens School], similar to gymnasium but without Latin or Greek.

We focus on some highlights of part A. Bunt motivated the reasons for choosing statistics as follows: to students in university disciplines such as economy, psychology and sociology, an extensive study of algebra is less useful than a well-balanced treatment of the first concepts and principles of statistics. Statistics in university turns out to be very difficult and uncommon to these students. Moreover, they have to learn it in a rather short period of time. Statistics in secondary school is not only useful for the aforementioned students, but for all citizens in modern society. By reducing the algebra content, Bunt found the necessary 35 classroom hours for his statistics course. After that, he justified the chosen topics. In the first experimental text, these topics were: frequency distribution, histogram, frequency curve, cumulative frequency, average, median, quartiles, range, mean deviation, standard deviation, quartile distance, permutations, variations (without repetitions), combinations, Pascal's triangle, Newton's binomial formula, some simple theorems from probability calculus, the binomial distribution for $p = 0.5$, the normal curve as a limit of the histogram of the binomial distribution (graphical, not with formulae). At the end of the course, some applications of the normal curve for calculating probabilities were presented. Linear regression and correlation were left out, because of being too time-consuming. Especially on the insistence of his cooperators, Bunt drastically changed the end by including a final chapter on hypothesis testing: estimating some characteristics of a population on the basis of a sample.

Bunt extensively deals with the principles of probability calculus for which he presents an axiomatic approach. Probability is a function that assigns to an event a number in the interval [0,1]. He starts from the following two axioms: (1) If $p \rightarrow \neg q$, then $P(p \text{ or } q) = P(p) + P(q)$; (2) If $p$ is the sure event, then $P(p) = 1$. From these axioms Bunt derives the complement and product rule. He illustrates these rules with examples about rolling dice. In the textbook, however, Bunt introduces the concept of probability differently. There he starts with the definition of Laplace: the probability of an event is the number of outcomes favorable for that event, divided by the total number of outcomes (under the condition of mutually exclusive and equally likely outcomes). After having dealt with the complement, the sum and the product rule, he introduces "another" definition: if it turns out that in a large number of repetitions of an experiment, $n$, an event happens $k$ times, then we are convinced that every time we repeat this experiment a sufficient number of times, this event will happen in $k/n$ part of this number. We then state that the probability of that event equals $k/n$. For probabilities derived from that "new" definition, the complement, sum and product rule keep their validity. We note that Bunt's approach contrasts sharply with that of his contemporary Gustave Choquet, then president of the *International Commission for the Study and Improvement of Mathematics Teaching* (CIEAEM), who proposed at the 9th meeting of the CIEAEM a definition of probability based on the mathematical concept of measure (translated from French):

> In a set $U$, one chooses a family $F$ of subsets $E$, to each of which we attach a number $m(E)$, called the measure of $E$. These subsets have the following properties: their union and their intersection are again part of $F$, even if the number of $E$'s is infinite. In the case of probabilities, the set $U$ has measure $m(U) = 1$. Each element of $U$ represents a possible event: all favourable events constitute a subset $E$ with measure $m(E)$. The probability of the favourable event is given by $m(E)/m(U)$. (Carleer, 1955-6, pp. 63–64)

The difference between Bunt's and Choquet's approaches illustrates the debate during the mid-1950s between the mathematics-didacticians and the mathematics-structuralists on how statistics should be introduced at the secondary school level.

Because of its innovative character, we discuss in some detail how Bunt explained the concept and procedure of hypothesis testing. He wrote about this:

> On the basis of a sample of 10 marbles out of a box with 5000 white and 5000 red marbles the probabilities of 0, 1, 2, …, 8, 9, 10 red marbles in that sample are 0.001, 0.010, 0.044, 0.117, 0.205, 0.246, 0.205, 0.117, 0.044, 0.010, 0.001. It follows that in 1.1% of all samples of 10 marbles there are 0 or 1 red marbles, and even so, in 5.5%, there are 0, 1 or 2 red marbles. And, in 5.5% of all samples there are 8, 9 or 10 red marbles. And moreover, in 1.1% of all samples there are 9 or 10 red marbles. Now suppose that the fraction $p$ of red marbles is unknown and we take a sample of 10 marbles. We shall agree that if $p = 0.5$ and there are 0, 1, 9 or 10 red marbles in the sample, we shall reject the hypothesis $p = 0.5$. If the hypothesis $p = 0.5$ is right we have a risk of 1.1% + 1.1% = 2.2% that we, in spite of this, reject the hypothesis. More precisely, there is a probability of 1.1% that we reject the hypothesis $p = 0.5$ on the strength of too small (or too large) a number of red marbles. Because, in this connection, we, for the time being, do not want to risk a greater probability than 2.5%, we stick to the mentioned agreement. This agreement, therefore, conforms to the following conditions: (a) if $p = 0.5$, we risk, both for too small and for too large a number of red marbles in the sample, a probability of not more than 2.5% that we reject the hypothesis $p = 0.5$; (b) both for too small and for too large a number of red marbles this probability lies as close to 2.5% as possible. When we reject the hypothesis $p = 0.5$, we say the hypothesis $p = 0.5$ is rejected with an unreliability of not more than 5%. (Bunt, 1957, p. 12)

The fraction $v$ is introduced as the number of red marbles divided by the number of marbles in the sample and its values which are or are not thought contradictory to $p = 0.5$ are represented by, respectively, dots and circles on an axis (Figure 2).

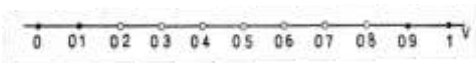


**Figure 2: Axis representing values of *v* with dots and circles**

Repeating this procedure for different values of $p$, one gets the two-dimensional scheme (Figure 3):

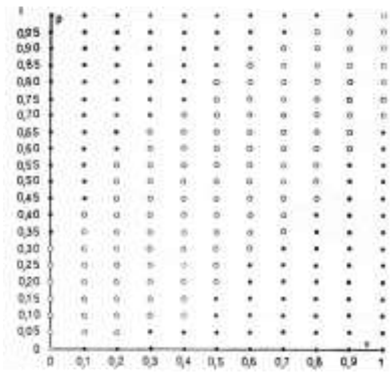


**Figure 3: *v*-axes for different values of *p***

By making the values of *v* and *p* "continuous", one gets a figure on which the different boundary lines refer to different sample sizes (Figure 4). The textbook contains two of these, corresponding to unreliabilities of 5% and 10%, called by Bunt "nomograms". From these nomograms, the student can observe that the probability of rejecting a false hypothesis increases with the sample size.

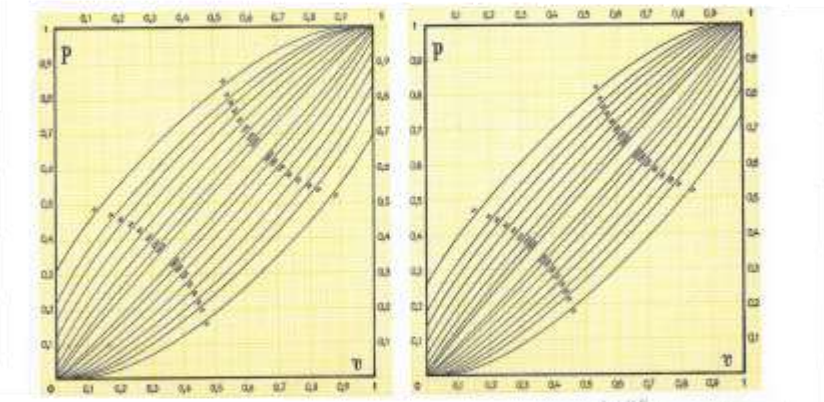

**Figure 4: Nomograms for different sample sizes (left, the unreliability is 5%, right 10%)**

## Consolidation and internationalization

In 1954-1955 a curricular commission of *Wimecos* published a report including a draft curriculum and central examination program for mathematics in HBS-B. Bunt had been a member of that commission representing the Dutch mathematics didacticians and mathematics teacher trainers. In the commission's report, it is stated that statistics had been important sources for the commission. The commission basically confirmed the conclusions of the report of *Liwenagel* (Liwenagel, 1950-1951), but now generalized to all students who prepared themselves for university studies. In 1958, the new curriculum was actually implemented, but, although it entailed a considerable change, statistics only became an optional subject for gymnasium A.

The fifth edition of Bunt's textbook (Bunt, 1968) had a slightly different title, a consequence of the curriculum reform consolidated in 1968 by a new law for secondary education. The subtitle, statistics for preparatory higher and secondary education, was changed into: statistics for preparatory scientific education. This new curriculum reform was prepared and supervised by the *Commissie Modernisering Leerplan Wiskunde* (CMLW) [Commission for Modernization of the Mathematics Curriculum]. The task of that commission was to prepare the mathematics curriculum reform in line with the ideas of Royaumont Seminar. Bunt was a member of the CMLW. The commission was officially set up in June 1961 by the Ministry of Education, Arts and Science, but already in January 1961, Bunt had proposed to the Ministry to establish such commission. However, the Inspection of Education had given a negative advice to the Ministry because the commission as proposed by Bunt was too small. In 1968 the new curriculum for mathematics, in which statistics played a clear role, was implemented in all schools for secondary education in the Netherlands: Bunt had achieved what he had started working on in 1951.

During the late 1950s and early 1960s, Bunt disseminated his ideas about the teaching of statistics. Already on May 24, 1959, he was invited to report on his experiment about the teaching of statistics at the annual meeting of the *Société Belge de Professeurs de Mathématiques* [Belgian Association of Mathematics Teachers] and the *Société Belge de Statistique* [Belgian Association of Statistics], held in Brussels on May 24, 1959 (Bunt, 1959). The manner in which statistics became a part of the secondary-school curriculum in the Netherlands was also the topic of Bunt's paper at the Royaumont Seminar (OEEC, 1961). In the period after Royaumont, Bunt had the opportunity to actively participate in meetings held in order to coordinate, monitor and refine the implementation of the Royaumont recommendations (Aarhus, 1960; Athens, 1963, Echternach, 1965).

## More recent developments

According to the law for secondary education of 1968, two types of schools could prepare students to higher education: *Voorbereidend Wetenschappelijk Onderwijs* (VWO, six grades for students from age 12 to 18) [preparatory scientific education], preparing for university studies, and *Hoger Algemeen Voortgezet Onderwijs* (HAVO, five grades for students from age 12 to 17) [higher general continued education]. The mathematics curricula of these school types were prepared by the CMLW. The curriculum of VWO included probability theory and statistics, that of HAVO only included descriptive statistics. These topics were meant to be taught in the last two years of these school types. We restrict ourselves to statistics teaching at VWO. Although Bunt's textbook was available, CMLW judged that it was better to not implement statistics immediately, but first to develop a new text and conduct an experiment with a restricted number of schools. The argument was that Bunt's textbook was only intended for students in the "old" gymnasia A, whereas statistics now had become a compulsory subject for all students. A statistics development team started in 1970, first under the supervision of the CMLW, from 1971 under the supervision of the then started IOWO, the predecessor of the Freudenthal Institute.

After a first draft the team developed the textbook (Nijdam et al., 1973) including the following content: Introduction, Probability rules, Probability distributions, Hypothesis testing and reliability intervals, Parameters of a distribution, Use of the normal distribution. The introduction contained an example with a prognosis of the number of students of VWO that should follow science or mathematics at the university, based on data of the Dutch *Central Bureau of Statistics*. From that example, terms as sample, population, random, representative, testing – for instance with respect to the quality of the production of certain items – were introduced. In this textbook the students themselves started with a probability experiment. There was a box with 1000 small marbles, 600 red and 400 black. With a kind of spoon with 20 wholes, they drew a random sample of 20 marbles. This box with the "spoon" was used to simulate various probability experiments.

## Conclusions

The mathematician Lucas Bunt played a crucial role in promoting and developing materials for statistics education at the secondary level, in the Netherlands but also at the international level. Indeed, in the post-Royaumont era, probability and statistics were seen as valuable elements of a worldwide reform of the mathematics curricula. Although Bunt explained his approach in a rather classical way, starting with some probability axioms (in pure *New Math* style), the approach in his textbook was very pragmatic. Bunt did not emphasize "theoretical aspects", accepted properties without proof and provided many clarifying examples. This pragmatic style enabled Bunt to explain the basic principles of hypothesis testing at the end of his course, in a limited number of lessons.

Nowadays in the Netherlands and in several other countries, probability and statistics are included in the mathematics programs, at least for some streams at the secondary level, but in the 1950s and 1960s, it was quite revolutionary to propose to teach these topics at that school level.

Because of his didactical work in general and more specifically on statistics, Bunt was important in Dutch mathematical education in the post-WWII period. Due to his participation to Royaumont and other international conferences, and his textbooks in English, Bunt may also have played a role in debates about the gradual introduction of statistical curricula for the secondary school level in other countries. However, this role has not yet been clarified and is a topic for follow-up research.

**Acknowledgements**

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